UNITED STATES DISTRICT COURT
NORTHERN DISTRICT OF CALIFORNIA
SAN FRANCISCO DIVISION

IN RE GOOGLE PLAY STORE ANTITRUST LITIGATION THIS DOCUMENT RELATES TO: *In re Google Play Consumer Antitrust Litigation*, Case No. 3:20-cv-05761-JD *State of Utah et al. v. Google LLC et al.*, Case No. 3:21-cv-05227-JD	No. 3:21-md-02981-JD **DECLARATION OF** **HAL J. SINGER, PH.D.** Judge: Hon. James Donato

INTRODUCTION

1. I have been asked by counsel for Consumer Plaintiffs to prepare this declaration in response to the Court's Order dated August 6, 2023 ("Order"), which posed four questions to me and Google's damages expert, Dr. Gregory Leonard.[1] My responses are detailed below. The economic literature and other materials cited herein are listed in Appendix 1, and the full text of each article is attached to this declaration.[2]

A. LOGIT IS STANDARD AND RELIABLE FOR MEASURING DEMAND AND PASS-THROUGH HERE

2. In this Section, I respond to Question A of the Order, which has two sub-parts.[3] Due to the nature of my response and the logical flow of the economic proof, I respond to sub-part (ii) first.

(ii) Logit Reliably Measures Demand Here

3. Question A, sub-part (ii) of the Order poses the following question:

> *What economic literature states that a regression analysis is a reliable way of… confirming that a logit model can be used to reliably measure the relevant demand curve here?*[4]

4. **Summary Answer**: The economic literature shows that regression analysis can be used to confirm that the logit model is well-specified here, meaning that the demand curves observed in the Play Store transactional data have the shape of the logit demand curve.

***

5. The shape of the demand curve is relevant here because it allows an economist to infer the pass-through rate: When there is a market-wide decrease in costs, each firm will respond

---

1. Order Re Dr. Singer's Proposed Expert Testimony (August 6, 2023).
2. For textbooks, the front matter and pages cited are included.
3. *Id.* at 2 ("What economic literature states that a regression analysis is a reliable way of (i) testing for the IIA assumption in the logit model, or (ii) confirming that a logit model can be used to reliably measure the relevant demand curve here?")
4. *Id.*

by cutting its price by an amount that depends on the shape of the demand curve it faces.[5] In his academic writings, Dr. Leonard has observed that the shape of the demand curve allows one to infer the pass-through rate.[6]

6. The logit model is a standard economic model describing the shape of the demand curve. Regression analysis is a standard econometric tool for analyzing relationships between two or more economic variables.[7] One use of regression analysis is to determine whether or not a given economic model (such as a demand curve with a particular shape) is well-specified for a given data set—whether it "best describes the process under study,"[8] in a way that "make[s] good economic sense."[9] Economists can and do use regression analysis to confirm the shape of the demand curve, including the logit demand curve. For example, Gerakos & Syverson (2015) used regression analysis to confirm that the "commonly used logit demand model" was well-specified to their data set.[10]

---

5. *See, e.g.,* Nathan Miller, Marc Remer, & Gloria Sheu, *Using Cost Pass-Through To Calibrate Demand*, 118 ECONOMICS LETTERS 451 (2013) (Exhibit 1). *Id.* at 452-453 (calculating different pass-through rates for different demand curves, including linear demand and logit demand).

6. Jerry Hausman & Greg Leonard, *Efficiencies from the Consumer Viewpoint*, 17(3) GEORGE MASON LAW REVIEW 707, 723. (1999) (Exhibit 2).

7. JEFFREY WOOLDRIDGE, INTRODUCTORY ECONOMETRICS: A MODERN APPROACH 1 (Thompson 4th ed. 2009), [hereafter WOOLDRIDGE] (Exhibit 3) ("Econometrics is based upon the development of statistical methods for estimating economic relationships[.]"). *Id.* at iii (First twelve chapters of textbook have "Regression Analysis" in the main heading). *See also* Daniel L. Rubinfeld, *Reference Guide on Multiple Regression*, *in* REFERENCE MANUAL ON SCIENTIFIC EVIDENCE 303 (Federal Judicial Center, National Academies Press 2011) (Exhibit 4) ("Multiple regression analysis is a statistical tool for understanding the relationship between two or more variables.")

8. ROBERT PINDYK & DANIEL RUBINFELD, ECONOMETRIC MODELS AND ECONOMIC FORECASTS 184 (McGraw-Hill 1998) [hereafter, PINDYK & RUBINFELD (1998)] (Exhibit 5) (Chapter 7 covers "model specification," and explains that "researchers usually...attempt to find the [regression] specification which best describes the process under study.") Note that the entire chapter deals with regression models. *Id.* at vi (Table of Contents shows Chapter 7, titled "Instrumental Variables and Model Specification" under the broader heading, titled "Single-Equation Regression Models.")

9. DAMODAR GUJARATI & DAWN PORTER, BASIC ECONOMETRICS 468 (McGraw-Hill 2009) [hereafter, GUJARATI & PORTER (2009)] (Exhibit 6) (explaining that an economic model chosen for empirical analysis "must make good economic sense. For example, if Milton Friedman's permanent income hypothesis holds, the intercept value in the regression of permanent consumption on permanent income is expected to be zero.") *Id.* at 467 (Chapter 13 covers "Econometric Modeling: Model Specification and Diagnostic Testing").

10. Joseph Gerakos & Chad Syverson, *Competition in the Audit Market: Policy Implications* 53 JOURNAL OF ACCOUNTING RESEARCH 725, 735 (2015) [hereafter, Gerakos & Syverson (2015)] (Exhibit 7) ("The demand model

7. I performed regression analysis demonstrating that the logit demand curve describes the Play Store's transaction data extremely well, in a way that makes economic sense. Applied here, the standard logit demand curve used in the economic literature predicts that, when an App's price increases, the App's share within its Play Store category—the contours of competition as determined by Google and the developer itself—will decline in a specific manner given by the standard logit demand curve equation, as shown in, *e.g.*, Verboven & Van Dijk (2009).[11] I designed my regression model to test whether this standard logit demand curve equation fits the Play Store transaction data. Accordingly, I specified the regression model to measure the relationship between prices and market shares observed in the data.[12] In other words, consistent with best practices, my regression model is grounded in "a theory that describes the variables to be included in the study."[13] My regression results demonstrate that the standard logit equation makes extremely accurate predictions when it is used to predict each App's share within its Play Store category in the transactional data. In fact, the logit equation explains over 95 percent of the

---

outlined earlier is a form of the commonly used logit model. This framework is commonly used in the economics literature (and elsewhere, such as in marketing research)…The frequency and breadth of its application reflects its usefulness and flexibility.") The authors use various regressions to confirm that the logit demand model is well-specified to their data set. *Id.* Tables 5, 6, 7; Table A1. *See also* WOOLDRIDGE (Exhibit 3) at 553 (illustrating how regression analysis is used to "trace out the demand equation.") This is an illustration of an instrumental variable ("IV") regression. *Id.* at 552. *See also id.* at 510 (defining IV). I applied IV regressions to estimate demand here. Singer Merits Report ¶353.

11. Frank Verboven & Theon Van Dijk, *Cartel Damages Claims and the Passing-on Defense*, 57(3) JOURNAL OF INDUSTRIAL ECONOMICS 457, 488 (2009) [hereafter, Verboven & Van Dijk (2009)] (Exhibit 8). The market share for a given App is given by the equation below, in which s_i is the category share for App i, and p_i is the price of that App. An increase in p_i results in a decrease in s_i. (The variable p_k denotes the prices of other Apps in the category).

$$s_i(\mathbf{p}) = \frac{\exp(v_i - \alpha p_i)}{1 + \sum_{k=1}^{N} \exp(v_k - \alpha p_k)}$$

12. The dependent variable in my regression model is based on the market share of a given App. Specifying demand curves in which the dependent variable is expressed as market share (that is, quantity divided by market size) is standard practice in economics. *See, e.g.,* Verboven & Van Dijk (2009) (Exhibit 8), *supra*, at 488. *See also* Jonas Bjornerstedt & Frank Verboven, *Merger simulation with nested logit demand*, 14(3) STATA JOURNAL 511, 514 (2014) [hereafter, Bjornerstedt & Verboven (2014)] (Exhibit 9) (equation (3)).

13. Daniel L. Rubinfeld, *Reference Guide on Multiple Regression*, *in* REFERENCE MANUAL ON SCIENTIFIC EVIDENCE 303, 311 (Federal Judicial Center, National Academies Press 2011) (Exhibit 4) ("Ideally, a multiple regression analysis builds on a theory that describes the variables to be included in the study.")

variation in an App's share within its respective Play Store category.[14] Accordingly, the demand curve for Apps has the shape given by the logit demand curve.

8. The regression results also make economic sense: The regression coefficient estimates confirm that, when the price of an App goes up, its share declines in the manner predicted by the logit equation (and vice-versa).[15] Downward-sloping demand is an elementary economic principle—when things cost more, consumers buy less (and vice-versa).[16] The downward-sloping demand shape predicted by the logit equation holds true for *every one* of the Play Store categories, and is highly statistically significant for every category.[17]

9. To further determine whether the logit demand curve was well-specified, I used regression analysis to compare the logit demand curve against alternative demand specifications, and the results demonstrated that logit demand is the superior specification.[18] For example, when I ran regressions using the linear demand specification, the results were contrary to elementary economic principles: The linear regression coefficient estimates indicated that the demand curve was not significantly downward-sloping for many Play Store categories.[19] Dr. Leonard has provided no evidence that any other demand curve describes the Play Store data better. Based on his expert report, Dr. Leonard implies that the linear model might better describe the data,[20] but it does not.

---

14. Singer Merits Report Table 12 (*R*-squared statistic above 95 percent).
15. Singer Merits Report Table 12.
16. N. GREGORY MANKIW, PRINCIPLES OF MICROECONOMICS 40 (Cengage Learning 8th ed. 2018) [hereafter MANKIW] (Exhibit 10).
17. Singer Merits Report Table 12.
18. As explained above, it is standard practice in econometrics to compare alternative specifications to determine which best described the data. *See* PINDYK & RUBINFELD (1998) (Exhibit 5) at 184 ("researchers usually...attempt to find the [regression] specification which best describes the process under study.").
19. Singer Merits Reply Appendix 3. *Id.* ¶51, n. 96 ("[L]inear demand does not fit the data well. Contrary to economics, the price coefficients are statistically insignificant in 12 out of the 33 regressions.") As explained above, a well-specified regression model "must make good economic sense." GUJARATI & PORTER (2009) (Exhibit 6) at 468.
20. Leonard Report ¶67; ¶100; Fig. 10; Exhibit 8.

10. My conclusion that my model is well-specified is also grounded in record evidence on how the market for Apps operates. This evidence shows that the Play Store categories are designed and utilized specifically to classify similar Apps into similar categories.[21] Other economists have implemented logit demand models similarly. For example, economists have used the logit model to analyze a merger between two publishers, using pre-defined categories of literature such as humor or thriller. The European Commission judged the analysis "particularly robust."[22] Other economists have determined that logit can be reliably applied even to categories that are arguably broader. A published, peer-reviewed analysis of the European vitamins cartel applied the logit model to vitamin premixers.[23] Premixers include large companies selling a "comprehensive range" of products tailored to, e.g., different types of livestock globally.[24]

---

21. Singer Merits Reply ¶65.

22. Oliver Budzinski & Isabel Ruhmer, *Merger Simulation in Competition Policy: A Survey*, Philipps-University Marburg, Faculty of Business Administration and Economics, Marburg (2008) [hereafter, Budzinski & Ruhmer] (Exhibit 11), at 21. Available at: https://www.econstor.eu/handle/10419/30108, citing European Commission, Case No. Comp/M.2978, Lagardere/Natexis/VUP, (January 2004) at 179, n. 543. These economists used a nested logit model, meaning that categories of books (such as "humour, thriller or love story") were grouped together. Budzinski & Ruhmer (Exhibit 11), *supra*, at 21. My model in this case is equivalent to a nested logit model in which each Play Store category is its own nest, with the conservative assumption that there is zero substitutability across different nests. *See, e.g.,* Bjornerstedt & Verboven (2014) (Exhibit 9) at 514-515.

23. Verboven & Van Dijk (2009) (Exhibit 8), supra at 482, Table III. At the Hearing, Dr. Leonard claimed that Verboven & Van Dijk (2009), "just use logit as an illustration of their approach" Hearing Tr. 29:19-20, because the authors briefly discuss possibilities for "a more complete analysis" *Id.* 29:9-11. Here, I have performed a complete analysis, including regression analysis confirming that the logit model is well-specified. Elsewhere, one of the same authors has implemented a logit model using regression methods. *See* Frank Verboven, *International Price Discrimination in the European Car Market*, 27(2) RAND JOURNAL OF ECONOMICS 240, 251 (1996) (Exhibit 12) [hereafter Verboven (1996)] (explaining the econometric methods used to implement the logit model). *Id.* at 257 (reporting three-stage least squares ("3SLS") estimation results for logit model). Note that 3SLS is a particular type of regression. *See, e.g.,* https://www.stata.com/manuals/rreg3.pdf (Exhibit 13) (describing "reg3" command to implement 3SLS).

24. *See, e.g.,* https://right-frank.com/en/suppliers/frank-wright.htm (Exhibit 14) ("Some of the products that the Frank Wright company manufactures and supplies are: A *comprehensive range* of the most advanced feed premixes for *all livestock*.") (emphasis added).

(i) The Independence of Irrelevant Alternatives Property Is Reliably Established Here

11. Question A, sub-part (i) of the Order poses the following question:

What economic literature states that a regression analysis is a reliable way of (i) testing for the IIA assumption in the logit model…? [25]

12. **Summary Answer:** As explained above, I have confirmed using standard regression methods from the economic literature that the logit demand curve is well-specified here. The economic literature teaches that Independence of Irrelevant Alternatives ("IIA") property "can be interpreted as a natural outcome of a well-specified model."[26] Accordingly, IIA is reliably established here. The economic literature does not require further testing of IIA, which would not be reliable or informative here.[27]

***

13. IIA is a property of logit. Applied here, IIA implies that consumers will tend to substitute among different Apps within a given category in proportion to an Apps' share in that category ("proportional substitution" or "proportionate shifting").[28] Suppose the price of App *A* increases. To avoid the price hike, some consumers will switch to different Apps within the same category. Suppose further that App *B* is very popular, with a category share of 50 percent, and that App *C* is less popular, with a category share of just one percent. Under proportional substitution, these consumers are more likely to switch to the (more popular) App *B* than they are to switch to

---

25. Order at 2.

26. Kenneth Train, *Logit, in* DISCRETE CHOICE METHODS WITH SIMULATION 34, 35 (Cambridge University Press 2009) [hereafter, TRAIN] (Exhibit 15).

27. There are additional methods that, in theory, can be used implement direct tests of IIA, but they have well-known practical limitations, and no expert has implemented them here. *See* n. 36, *infra*. Given the limitations of such tests, economists frequently use logit without implementing them. *See* n. 37, *infra*.

28. TRAIN (Exhibit 15), *supra*, at 47 ("This pattern of substitution, which can be called *proportionate shifting*, is a manifestation of the IIA property.") (emphasis in original). *See also* Nathan Miller & Gloria Sheu, *Quantitative Methods for Evaluating the Unilateral Effects of Mergers*, 58 REVIEW OF INDUSTRIAL ORGANIZATION 143, 171 (2021) [hereafter, Miller & Sheu (2021)] (Exhibit 16) ("Demand is logit…This implies that diversion is proportional to market share, in the sense that the relative diversion from any product i to any two other products k and j takes the form: $d_{ij}/d_{ik} = s_j/s_k$. This property is also known as the 'independence of irrelevant alternatives' (IIA).")

the (less popular) App *C*. Specifically, consumers are, on average, fifty times more likely to switch to App *B* than App *C* under this assumption.

14. As Professor Train explains in his authoritative graduate-level textbook, the IIA property "can be interpreted as a natural outcome of a well-specified model."[29] Accordingly, "[i]n many settings, choice probabilities that exhibit IIA provide an accurate representation of reality."[30] That logit frequently provides an accurate representation of reality is confirmed by its widespread use by applied economists,[31] including to estimate pass-through.[32] As explained above, I have confirmed using standard regression methods that the logit demand curve is well-specified here. Therefore, proportional substitution can be reliably inferred here.

15. The intuition for this result is as follows: If a regression model is well-specified, then it will accurately predict category shares; anything not predicted by the model is inherently unpredictable and therefore random ("white noise").[33] To extend the prior example: A well-specified logit model will accurately predict that App *B* is fifty times more popular than App *C*—

---

29. TRAIN (Exhibit 15), *supra,* at 35.

30. TRAIN (Exhibit 15), *supra,* at 46.

31. *See, e.g.,* Gerakos & Syverson (2015) (Exhibit 7) at 735 ("The demand model outlined earlier is a form of the commonly used logit model. This framework is commonly used in the economics literature (and elsewhere, such as in marketing research)…The frequency and breadth of its application reflects its usefulness and flexibility."); *see also* Luke Froeb et al., *Economics at the Antitrust Division: 2017–2018*, 53 REVIEW OF INDUSTRIAL ORGANIZATION 637 (2018) [hereafter, Froeb et al. (2018)] (Exhibit 17); Verboven & Van Dijk (2009) (Exhibit 8); Verboven (1996) (Exhibit 12); David Besanko, Sachin Gupta, & Dipak Jain, *Logit Demand Estimation Under Competitive Pricing Behavior: An Equilibrium Framework*, 44 MANAGEMENT SCIENCE 1533 (1998) [hereafter, Besanko et Al. (1998)] (Exhibit 18). *See also* Singer Merits Report ¶348, n. 809. Google's class certification economist, Dr. Michelle M. Burtis, conceded that logit is "frequently used in economics[.]" Expert Report of Dr. Michelle M. Burtis dated March 31, 2022 ("Burtis Class Report") ¶306.

32. *See, e.g.,* K. Sudhir, *Structural Analysis of Manufacturer Pricing in the Presence of a Strategic Retailer* 20(3) MARKETING SCIENCE 244-264 (2001) (Exhibit 19) (using logit to analyze pass-through of wholesale supermarket prices into retail prices paid by consumers); David Besanko, Jean-Pierre Dubé & Sachin Gupta, *Own-Brand and Cross-Brand Retail Pass-Through*, MARKETING SCIENCE 123, 127 (2005) (Exhibit 20) (Table 1 summarizes logit pass-through rates); Nathan Miller, Marc Remer, Conor Ryan, & Gloria Sheu, *Pass-Through and the Prediction of Merger Price Effects*, 64(4) THE JOURNAL OF INDUSTRIAL ECONOMICS 683-709, 693 (2016) (Exhibit 21) (Table 1 shows pass-through estimates for logit); Verboven & Van Dijk (2009) (Exhibit 8) at 457 (using logit to analyze the extent to which direct purchasers overcharged by the European Vitamins Cartel would pass on the overcharges to indirect purchasers).

33. TRAIN (Exhibit 15), *supra,* at 35.

that is, it will accurately predict the Apps' shares, and thus their shares relative to one another, and thus how much more consumers prefer one to the other on average. It follows that the model will accurately predict that consumers are fifty times more likely to switch to App *B* than App *C*. Any errors in this prediction are random; hence, the prediction is correct on average.

16. There is no evidence that IIA is not satisfied here. Dr. Leonard's cherry-picked examples of four seemingly disparate Apps—plucked from hundreds of thousands of SKUs in the Play Store transaction data—which purport to demonstrate that IIA is violated,[34] have no basis in standard economic practice. There are millions of data points in the Play Store transaction data.[35] No economic model could predict all of them with 100 percent accuracy, nor is this necessary: What matters is that the logit's predictions are systematically accurate on average, as explained above.

17. There are methods that, in theory, can implement direct tests of IIA, but they have well-known practical limitations.[36] No expert has implemented them here. The economic literature demonstrates that economists frequently use the logit demand curve without implementing any such test of IIA.[37]

---

34. Dr. Gregory K. Leonard, Slides for Merits Expert Proceeding (August 1, 2023) [hereafter, Leonard Deck] at 55-56.

35. Singer Merits Report Table 12 (showing more than 80 million observations used in logit regression).

36. Academic economists have developed models that can theoretically test IIA. These techniques suffer from well-known practical problems, which can severely limit their applicability and reliability when applied to real-world data sets. *See*, *e.g.* Christopher Knittel & Konstantinos Metaxoglou, *Estimation Of Random-Coefficient Demand Models: Two Empiricists' Perspective* 96(1) REVIEW OF ECONOMICS AND STATISTICS 34 (2014) (Exhibit 22). Other tests are even "more difficult to perform," and, even if they can be performed, "these tests do not provide as much guidance on the correct specification to use instead of logit." TRAIN (Exhibit 15), *supra*, at 50.

37. *See, e.g.*, Froeb et al. (2018) (Exhibit 17) 637 (IIA tests not mentioned in discussion of DOJ economists' use of logit, nor included in the accompanying 98-page antitrust software manual, available at: https://cran.r-project.org/web/packages/antitrust/antitrust.pdf); *see also* Verboven & Van Dijk (2009) (Exhibit 8), *supra*, at 457-91 (IIA tests not mentioned in article using logit to measure pass-through from cartel, nor is it stated that these tests should be performed in a more extensive analysis); Verboven (1996) (Exhibit 12) 240 (IIA tests not used in logit analysis of European auto pricing). *See also* Besanko et al. (1998) (Exhibit 18) (IIA tests not used in an article applying logit demand to retail scanner data).

B. LOGIT WOULD STILL BE A RELIABLE APPROXIMATION HERE EVEN IF IIA WERE NOT STRICTLY SATISFIED

18. Question B of the Order poses the following questions:

> *To what extent can IIA be 'not strictly satisfied' before the use of logit model becomes unreliable? How can the Court know that this limit has not been crossed here? How close is the "approximation" that Dr. Singer posits, and how can the Court have confidence that his logit model has produced a sufficiently reliable approximation of pass-through here even if the apps in each category are not proportional substitutes for one another?*[38]

19. **Summary Answer:** As explained above, the economic literature and my empirical analysis establish that the demand curve for Apps has the logit shape, and that proportional substitution (that is, IIA), holds. It follows that the logit pass-through rates that I have calculated are reliable. But even if one were to assume, counterfactually, that there were evidence that IIA had been violated here, pass-through could still be estimated reliably using the logit pass-through formula, because logit would provide an "adequate approximation to reality."[39] The reliability of IIA as an approximation depends on "the goals of the research."[40] If my goal here were to forecast large shifts in demand (such as the demand that would materialize for a new product), a violation of proportional substitution could render this forecast unreliable. In contrast, the purpose of my analysis here—to estimate pass-through to consumers *while holding market shares constant*—would make logit appropriate to use as an approximation even if I believed that IIA were not strictly satisfied.

---

38. Order at 2.

39. TRAIN (Exhibit 15), *supra,* at 48 ("when IIA reflects reality (or an adequate approximation to reality), considerable advantages are gained by its employment.") Prof. Rysman, who I understand has never studied my model in this case, made a similar observation independently in his deposition. *See* Rysman Tr. 65:19-66:11 ("Q. Would any reasonable economist, in your view, apply a standard logit model where the IIA property does not hold?...A. Yes. Q. When would it be appropriate to apply a standard logit model where IIA does not hold?...A. Well, every economic model is an approximation of -- of reality…But the question for economists isn't to, sort of, get the model to match the real world exactly but to capture the main features and be able to answer the questions that you want to answer in a compelling way.")

40. TRAIN (Exhibit 15), *supra*, at 36. Again, Prof. Rysman, made a similar observation in his deposition. Rysman Tr. 65:7-9 ("Q. In what applications can IIA be problematic? A. It depends on the question at hand.")

***

20. Some applications of logit rely on proportional substitution to forecast the extent to which market shares would change after the introduction of a new product. This is commonly illustrated using the classic "red bus, blue bus" example.[41] Suppose commuters initially have a choice of (1) going to work by car; or (2) taking a blue bus to work. Suppose that half select each option. Next, suppose that a red bus is introduced, which is identical to the blue bus (except in color) so that commuters now have three options (red bus, blue bus, or car). Under proportional substitution, a naïvely specified logit model would predict that one third of commuters would take the red bus, one third would take the blue bus, and one third would travel by car. This is a very substantial shift in market shares: Previously, only half of commuters took the bus; now, two-thirds do so. Thus, in this stylized example, proportional substitution would tend to over-forecast future demand for travel by bus.

21. My analysis requires no such forecast: In calculating pass-through rates, no new products are introduced to the market, and there is no shift in developers' category shares in the but-for world. To the contrary, developers' category shares are held fixed in the but-for world, as all developers lower their prices in response to a market-wide decrease in costs. My logit pass-through rates therefore do not hinge on the IIA property of proportional substitution to forecast *any* shift in market share. Because market shares are unchanged in the but-for world, what matters for the accuracy of my pass-through rates is that the logit demand curve accurately predicts the *actual* market shares observed in the Play Store transaction data. In econometric terms, what matters for the accuracy of my pass-through rates is that the regression performs extremely

41. *See, e.g.,* TRAIN (Exhibit 15), *supra*, at 46.

accurate "in-sample forecasting."[42] Therefore, my logit pass-through rates would still be reliable, even if one were to assume counterfactually that the IIA property of proportional substitution is not strictly satisfied here.

C. THE PER-UNIT PASS THROUGH RATE IS NOT NECESSARILY GREATER THAN THE *AD VALOREM* PASS-THROUGH RATE HERE

22. Question C of the Order poses the following questions:

> *Is a per-unit pass-through rate necessarily greater that an ad valorem pass-through rate in this case? If so, why? Does this affect the reliability of Dr. Singer's pass-through formula?*[43]

23. **Summary Answer**: A per-unit pass-through rate is not distinguishable from an *ad valorem* pass-through rate in this case, because Apps are competitively supplied and not monopolized. In a highly competitive market, such as the one I model here, there is no economically significant difference between the pass-through of cost savings flowing from a lower *ad valorem* take rate, versus cost savings flowing from lower per-unit costs.

***

24. I use the pass-through formula derived in Miller et. al. (2013), which solves for the pass-through rate in competitively supplied industries under a general market-wide change in firms' marginal costs.[44] The authors' "General model of pass-through"[45] yields a general formula for the logit pass-through rate that is not specific to per-unit costs or *ad valorem* costs, but instead

---

42. GUJARATI & PORTER (2009) (Exhibit 6) at 493 ("In-sample forecasting essentially tells us how well the model fits the data in a given sample."). The *R*-squared statistic measures how accurately the model performs for in-sample forecasts. *Id.* The *R*-squared for the regression is above 95 percent, meaning that logit demand explains nearly all of the variation in the actual market shares observed in the transaction data. Singer Merits Report Table 12. (*R*-squared statistic above 95 percent).
43. Order at 2.
44. Miller et. al. (2013) (Exhibit 1), *supra*, at 452.
45. Miller et. al. (2013) (Exhibit 1), *supra*, at 452.

turns on the change in the firms' marginal costs.[46] When applied to this case, the logit pass-through rate is equal to one minus a given developer's share in a given category. For example, if a developer's category share is five percent, the developer's pass-through rate is 95 percent. For every dollar in cost savings that the developer enjoys as a result of a lower take rate, 95 cents will be passed on to consumers in the form of lower prices.[47] The amount passed on to consumers is the same, regardless of whether the $1 in marginal cost savings resulted from a $1 decrease in per-unit marginal costs, or from a $1 in marginal cost savings resulting from a lower *ad valorem* take rate. What matters is the *change* in marginal cost, regardless of whether it comes from a change in per-unit costs or *ad valorem* costs.[48]

25. Dr. Leonard has presented his own set of calculations for this case that, he claims, should be used here instead of Miller et. al. (2013).[49] Dr. Leonard is wrong. Dr. Leonard's calculations are unsupported by any economic literature and are not applicable here. Dr. Leonard's

---

46. Miller et. al. (2013) (Exhibit 1), *supra*, at 452. The authors introduce a "per-unit tax" as an illustrative mathematical placeholder—solely for purposes of "perturb[ing] marginal costs and allow[ing] for the derivation of cost pass-through." *Id.* Nowhere to the authors state that the pass-through rate they derive would be any different in the case of an *ad valorem* cost.

47. Miller et. al. (2013) (Exhibit 1), *supra*, at 453. The logit pass-through rate is obtained by multiplying the right-hand side of equation (6) by negative one and inverting it, which yields $[M - Q]/M$, where M is the size of the category and Q is the quantity sold of a given App. For example, suppose that a developer accounts for five percent of a category. This means that Q/M = 0.05. The firm's pass-through rate is [M – Q]/M = M/M – Q/M = 1 – Q/M = 1 – 0.05 = 0.95. Thus, in this example, for each one-dollar increase in costs, the developer will increase prices to consumers by $0.95. Prior to Miller et. al. (2013), Professor Sudhir of Yale had previously derived a similar (albeit more limited) result in an article published in 2001. Prof. Sudhir restricted his analysis to a single retailer (whereas Miller et. al. derived more general, industrywide formulae). *See,* K. Sudhir, *Structural Analysis of Manufacturer Pricing in the Presence of a Strategic Retailer* 20(3) MARKETING SCIENCE 244-264 (2001) (Exhibit 19) (using logit to analyze pass-through of wholesale supermarket prices into retail prices paid by consumers). *Id.* at 251, equation (12) (with logit demand, a profit-maximizing retailer will, in response to a change in the wholesale price for a given product, adjust the retail price of that product by an amount proportional to $(1 - S_I)$, where S_I is the share of the product in question).

48. This principle is also illustrated in Ganapati et. al. (2020), which demonstrate that, when a percentage tax rate is imposed, it is the *change* in cost that determines the amount passed on to consumers. *See* Sharat Ganapati, Joseph Shapiro, & Reed Walker, *Energy Cost Pass-Through in US Manufacturing: Estimates and Implications for Carbon Taxes* 12(2) AMERICAN ECONOMIC JOURNAL: APPLIED ECONOMICS 303, 312-315 (2020) (Exhibit 23). Note that a percentage tax rate is the same as an *ad valorem* tax.

49. Leonard Damages Report Appendix D. Dr. Leonard admitted in his deposition that, even if one wanted to apply his pass-through formula here, it would be impossible because it would require data on every individual developer's marginal costs, and these data are not available. Leonard Dep. at 87:13-88:7.

calculations assume that each developer is monopolist, which is flatly inconsistent with the facts of this case.[50] In contrast, Miller et. al. (2013) model pass-through in a competitive market, as is appropriate for this case.[51]

26. Dr. Leonard's submission of Suits and Musgrave (1953) at the hearing on August 1, 2023[52] does not upset my conclusions, and in fact supports them. Based on this article, Dr. Leonard claims "pass-through rate for a per unit tax exceeds the pass-through rate for an ad valorem tax."[53] But as Dr. Leonard admits on the very same slide, this finding applies only "*under monopoly*."[54] And even in the case of monopoly, the result is far from definitive: Suits and Musgrave (1953) show only that a monopolist's pass-through for a per unit tax exceeds pass-through for an *ad valorem* tax when the monopolist faces a linear demand curve.[55] When demand is not linear, whether or not the monopolist's per-unit pass-through exceeds the ad valorem pass-through depends on details of the market, such as "cost and demand conditions."[56] Thus, even if developers in the Play Store were monopolists (which they are not), an economist still could not necessarily conclude that the *ad valorem* pass-through rate would exceed the per-unit pass-through rate.

---

50. Dr. Leonard performs his analysis by solving a profit maximization equation for a *single developer*. Leonard Damages Report Appendix D, ¶5 (showing a *single* developer's profit function in equations A.1 and A.2).

51. Unlike Dr. Leonard, Miller et. al. (2013) simultaneously solve profit maximization problems for multiple firms operating in a competition with one another. *See* Miller et. al. (2013) (Exhibit 1) at 452 (solving for an expression involving the "cost pass-through matrix." Matrix algebra is necessary to solve multiple equations simultaneously. Multiple equations are necessary because multiple firms are competing in the market).

52. Leonard Deck at 48 (citing D. B. Suits & R. A. Musgrave, *Ad valorem and Unit Taxes Compared*, 67(4) QUARTERLY JOURNAL OF ECONOMICS 598 (1953) [hereafter, Suits & Musgrave (1953)] (Exhibit 24)).

53. Leonard Deck at 48.

54. Leonard Deck at 48 (emphasis added).

55. Suits & Musgrave (1953) (Exhibit 24) at 603 ("for the linear case… the final price under the unit tax will *always* be higher.") (emphasis in original).

56. *Id.* at 603.

27. More fundamentally, Dr. Leonard failed to disclose in his slides that Suits and Musgrave (1953) reach "precisely the opposite"[57] conclusion when they consider the opposite case of pure competition. Under pure competition, there is no difference between per-unit pass-through and *ad valorem* pass-through.[58]

28. The App categories at issue here are far closer to competition than monopoly. This can be seen in the pass-through rates I have calculated, which directly measure concentration within Play Store categories: The pass-through rate is equal to one minus the developer's category share. If App categories were monopolistic (or close to it), I would have calculated pass-through rates at or near zero. In fact, the weighted average pass-through rate is 91 percent[59]—close to the purely competitive pass-through rate of 100 percent. This 91 percent weighted average pass-through rate means that the weighted average developer category share is roughly nine percent (equal to 100 less 91). Again, this is nowhere near monopoly.

29. Dr. Leonard's submission of Adachi & Fabinger (2022) at the hearing on August 1, 2023 does not upset my conclusions.[60] Based on this paper, Dr. Leonard claims that "the pass-through rate for a per unit tax will exceed the pass-through rate for an ad valorem tax under oligopoly."[61] This result simply does not apply here. Unlike Miller et. al. (2013), Adachi & Fabinger (2022) describe their paper as contributing to the economic literature focusing on

---

57. *Id.* at 604.

58. *Id.* at 598 ("Under pure competition it is obvious that unit and *ad valorem* taxes which result in equal yields will also result in equal final prices.") If tax yields are not equal, then the *ad valorem* pass-through rate *exceeds* the per-unit pass-through rate under competition. Suits & Musgrave (1953) (Exhibit 24) at 602. This result is not relevant here because I effectively hold the yield fixed by calculating damages by multiplying developers' dollar savings from a lower take rate by the pass-through rate. To extend the example, my method would use the dollar in sales tax as the basis for damages, which would be the same for a per-unit tax and an *ad valorem* tax.

59. Singer Merits Report Table 13.

60. Leonard Deck at 49, citing Takanori Adachi & Michal Fabinger, *Pass-Through, Welfare, and Incidence Under Imperfect Competition* 211 JOURNAL OF PUBLIC ECONOMICS 1 (2022) [hereafter, Adachi & Fabinger (2022)] (Exhibit 25).

61. Leonard Deck at 49.

"industries [that] are characterized as oligopolies where a small number of firms are dominant."[62] As explained above, that does not describe the App market; developers operate in a competitive market. And unlike Miller et. al. (2013), Adachi & Fabinger (2022) study changes to multiple industrywide costs simultaneously—they do not separately study the effect of an ad valorem tax by itself, versus a per-unit tax by itself.[63] Adachi & Fabinger (2022) generate more than 400 equations,[64] yet Dr. Leonard's formula for the monopoly pass-through rate that he calculated for this case does not appear anywhere in Adachi & Fabinger (2022).

D. THE AT&T BENCHMARK IS RELIABLE AND CONSERVATIVE HERE

30. Question D of the Order poses the following question:

> *For the Play Points analysis, Dr. Singer used the example of AT&T in the 1980s as a benchmark for modeling the but-for world, positing that Google's market share would likely decline to 60% with competition, just as AT&T's did decades ago. Is the AT&T case an example of a two-sided market like the Google Play Store? Does that matter for its appropriateness for use as a benchmark here?* [65]

31. **Summary Answer**: AT&T's telephone network can be characterized as operating in a two-sided market: "[T]elephone services, for which most users are both callers and receivers, cannot be treated as one-sided markets."[66] AT&T's share loss post-divestiture is an appropriate benchmark for Google absent the Challenged Conduct regardless of whether AT&T's telephone network is characterized as a two-sided market or a one-sided market. In either case, the market

62. Adachi & Fabinger (2022) (Exhibit 25) at 1.
63. *Id.* at 3 ("this paper provides a generalization of Weyl and Fabinger's (2013) model to encompass the case of *multi-dimensional* interventions…) (emphasis in original).
64. Adachi & Fabinger (2022) (Exhibit 25) (over 400 equations appear throughout the article).
65. Order at 2.
66. Jean-Charles Rochet & Jean Tirole, *Platform Competition in Two-Sided Markets*, 1(4) EUROPEAN ECONOMIC ASSOCIATION 990, 1018 n. 31 (2003) [hereafter, Rochet & Tirole (2003)] (Exhibit 26).

for telephone services is characterized by "network effects," which confer a substantial incumbency advantage to an entrenched monopolist.[67]

***

32. Two-sided markets are characterized by what economists call "indirect network effects," meaning that the value of the platform to the users on one side is higher when there are more users on the other side of the platform.[68] If a market is one-sided, it may still exhibit "direct network effects," which means that "the value of a product, service, or platform increases simply because the number of users increases[.]"[69]A market with network effects confers a substantial incumbency advantage to an entrenched monopolist, regardless of whether it is one-sided or two-sided.[70] Regardless of whether it is characterized as one-sided or two-sided, telephone networks exhibit network effects: The larger the telephone network, the more valuable it is to users.[71]

33. Using the AT&T benchmark to model Google's share absent the Challenged Conduct is conservative because AT&T still enjoyed a substantial incumbency advantage flowing from network effects after it was compelled to operate with independent long-distance providers.

---

67. Jesse Romero, *Network Effects,* ECON FOCUS (Federal Reserve Bank of Richmond 2018) [hereafter, Romero (2018)] (Exhibit 27) ("Network effects can contribute to a situation known as "lock in," in which a particular standard becomes dominant and consumers find it very costly to switch. In these situations, the producer of the standard may be able to exercise monopoly power."). *See also* Tim Stobierski, *What Are Network Effects?* Harvard Business School Online (2020) [hereafter Stobierski (2020)] (Exhibit 28) ("'Once you've gained significant market share, you can often sit back and let the network effect take over[.]").

68. *See, e.g.,* David Evans, *Two-Sided Market Definition* in MARKET DEFINITION IN ANTITRUST: THEORY AND CASE STUDIES (ABA Section of Antitrust Law) 1-35, 5 (2009), papers.ssrn.com/sol3/papers.cfm?abstract_id=1396751 (Exhibit 29) ("A key feature of two-sided platforms is the presence of 'indirect network effects.'").

69. Stobierski (2020) (Exhibit 28).

70. Romero (2018) (Exhibit 27) (network effects allow for "lock-in" and the ability to exercise monopoly power"); *see also* Stobierski (2020) (Exhibit 28) ("'Once you've gained significant market share, you can often sit back and let the network effect take over[.]").

71. Romero (2018) (Exhibit 27) ("Telephones exhibit direct network effects[.]") *See also* Simran Kahai, David Kaserman & John Mayo, *Is the "Dominant Firm" Dominant? An Empirical Analysis of AT&T'S Market Power*, 39 JOURNAL OF LAW & ECONOMICS 499, 507 (1996) (Exhibit 30) ("We allow for a nonlinear (quadratic) relationship between subscribership and demand due to the network characteristic of telecommunications consumption-a doubling of subscribership is likely to more than double the market demand.") *Id.* at 509 ("This result is consistent with theoretical expectations for products subject to network externality effects.")

When I examined other industries with network effects, I found that dominant incumbents lost more share than AT&T when competition was introduced.[72] Netflix is a streaming service that brings together content creators and consumers. Netflix benefits from network effects: As more users join the platform, the platform can provide more and better content to users.[73] Netflix has seen its share fall by *more* than AT&T (from 90 percent to 25 percent).[74] In the market for Internet browsers, which also has network effects,[75] Internet Explorer's market share has declined from 95 percent to 54 percent.[76] Thus, my use of AT&T as the competitive benchmark was conservative.

72. Singer Merits Reply ¶¶43-46.

73. *See, e.g., Netflix is gaining subscribers again – but here's how it can succeed once people stop signing up*, THE CONVERSATION, July 20, 2023, *available at* https://theconversation.com/netflix-is-gaining-subscribers-again-but-heres-how-it-can-succeed-once-people-stop-signing-up-204717#:~:text=Network%20effects,-Founded%20in%201997&text=As%20more%20users%20joined%20the,launch%20%E2%80%93%20well%20before%20potential%20competitors (Exhibit 31).

74. Singer Merits Reply ¶44.

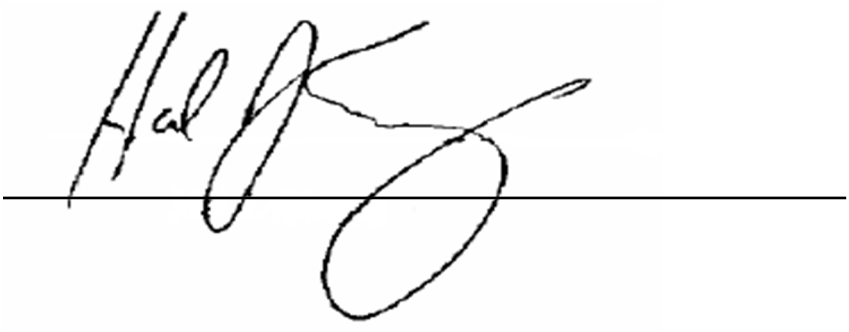
75. Romero (2018) (Exhibit 27).

76. Singer Merits Reply ¶46.

* * *

I declare under penalty of perjury under the laws of the United States of America that the foregoing is true and correct. Executed on this 14th day of August, 2023 at Vienna, Virginia.

Hal J. Singer, Ph.D.:



APPENDIX 1: LITERATURE CITED

Exhibit	Article or Textbook
1	Nathan Miller, Marc Remer, & Gloria Sheu, *Using Cost Pass-Through To Calibrate Demand*, 118 ECONOMICS LETTERS 451-453 (2013).
2	Jerry Hausman & Greg Leonard, *Efficiencies from the Consumer Viewpoint*, 17(3) GEORGE MASON LAW REVIEW 707, 723 (1999)
3	JEFFREY WOOLDRIDGE, INTRODUCTORY ECONOMETRICS: A MODERN APPROACH 1 (Thompson 4th ed. 2009)
4	Daniel L. Rubinfeld, *Reference Guide on Multiple Regression*, *in* REFERENCE MANUAL ON SCIENTIFIC EVIDENCE 303, 311 (Federal Judicial Center, National Academies Press 2011)
5	ROBERT PINDYK & DANIEL RUBINFELD, ECONOMETRIC MODELS AND ECONOMIC FORECASTS 184 (McGraw-Hill 1998)
6	DAMODAR GUIJARATI & DAWN PORTER, BASIC ECONOMETRICS 468 (McGraw-Hill 2009)
7	Joseph Gerakos & Chad Syverson, *Competition in the Audit Market: Policy Implications* 53 JOURNAL OF ACCOUNTING RESEARCH 725, 735 (2015)
8	Frank Verboven & Theon Van Dijk, *Cartel Damages Claims and the Passing-on Defense*, 57(3) JOURNAL OF INDUSTRIAL ECONOMICS 457, 481, 482, 487, 491 (2009)
9	Jonas Bjornerstedt & Frank Verboven, *Merger simulation with nested logit demand*, 14(3) STATA JOURNAL 511, 514-515 (2014)
10	N. GREGORY MANKIW, PRINCIPLES OF MICROECONOMICS 40 (Cengage Learning 8th ed. 2018)
11	Oliver Budzinski & Isabel Ruhmer, *Merger Simulation in Competition Policy: A Survey*, Philipps-University Marburg, Faculty of Business Administration and Economics, Marburg (2008), available at: https://www.econstor.eu/handle/10419/30108
12	Frank Verboven, *International Price Discrimination in the European Car Market*, 27(2) RAND JOURNAL OF ECONOMICS 240 (1996)
13	STATA Manual, *reg3, available at* https://www.stata.com/manuals/rreg3.pdf.

Exhibit	Article or Textbook
14	*Frank Wright*, *available at* https://right-frank.com/en/suppliers/frank-wright.htm
15	Kenneth Train, *Logit, in* DISCRETE CHOICE METHODS WITH SIMULATION 34 (Cambridge University Press 2009)
16	Nathan Miller & Gloria Sheu, *Quantitative Methods for Evaluating the Unilateral Effects of Mergers*, 58 REVIEW OF INDUSTRIAL ORGANIZATION 143, 171 (2021)
17	Luke Froeb et al., *Economics at the Antitrust Division: 2017–2018*, 53 REVIEW OF INDUSTRIAL ORGANIZATION 637 (2018)
18	David Besanko, Sachin Gupta, & Dipak Jain, *Logit Demand Estimation Under Competitive Pricing Behavior: An Equilibrium Framework*, 44 MANAGEMENT SCIENCE 1533 (1998)
19	K. Sudhir, *Structural Analysis of Manufacturer Pricing in the Presence of a Strategic Retailer* 20(3) MARKETING SCIENCE 244 (2001)
20	David Besanko, Jean-Pierre Dubé & Sachin Gupta, *Own-Brand and Cross-Brand Retail Pass-Through*, MARKETING SCIENCE 123 (2005)
21	Nathan Miller, Marc Remer, Conor Ryan, & Gloria Sheu, *Pass-Through and the Prediction of Merger Price Effects*, 64(4) THE JOURNAL OF INDUSTRIAL ECONOMICS 683 (2016)
22	Christopher Knittel & Konstantinos Metaxoglou, *Estimation Of Random-Coefficient Demand Models: Two Empiricists' Perspective* 96(1) REVIEW OF ECONOMICS AND STATISTICS 34 (2014)
23	Sharat Ganapati, Joseph Shapiro, & Reed Walker, *Energy Cost Pass-Through in US Manufacturing: Estimates and Implications for Carbon Taxes* 12(2) AMERICAN ECONOMIC JOURNAL: APPLIED ECONOMICS 303 (2020)
24	D. B. Suits & R. A. Musgrave, *Ad valorem and Unit Taxes Compared*, 67(4) QUARTERLY JOURNAL OF ECONOMICS 598 (1953)
25	Takanori Adachi & Michal Fabinger, *Pass-Through, Welfare, and Incidence Under Imperfect Competition* 211 JOURNAL OF PUBLIC ECONOMICS 1 (2022)
26	Jean-Charles Rochet & Jean Tirole, *Platform Competition in Two-Sided Markets*, 1(4) J. EUROPEAN ECONOMIC ASSOCIATION 990 (2003)

Exhibit	Article or Textbook
27	Jesse Romero, *Network Effects,* Econ Focus (Federal Reserve Bank of Richmond 2018)
28	Tim Stobierski, *What Are Network Effects?* Harvard Business School Online (2020)
29	David Evans, *Two-Sided Market Definition* IN MARKET DEFINITION IN ANTITRUST: THEORY AND CASE STUDIES (ABA Section of Antitrust Law) 1-35, 5 (2009), papers.ssrn.com/sol3/papers.cfm?abstract_id=1396751
30	Simran Kahai, David Kaserman & John Mayo, *Is the "Dominant Firm" Dominant? An Empirical Analysis of AT&T'S Market Power*, 39 Journal of Law & Economics 499, 507 (1996)
31	*Netflix is gaining subscribers again – but here's how it can succeed once people stop signing up,* The Conversation, (July 20, 2023), *available at* https://theconversation.com/netflix-is-gaining-subscribers-again-but-heres-how-it-can-succeed-once-people-stop-signing-up-204717#:~:text=Network%20effects,-Founded%20in%201997&text=As%20more%20users%20joined%20the,launch%20%E2%80%93%20well%20before%20potential%20competitors

E-FILING ATTESTATION

I, Lee M. Mason, am the ECF User whose ID and password are being used to file this document. In compliance with Civil Local Rule 5-1(h)(3), I hereby attest that each of the signatories identified above has concurred in this filing.

/s/ Lee M. Mason
Lee M. Mason